FDTD Analysis of Magnetized Ferrites: A More Efficient Algorithm

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Abstract— **A more efficient FDTD algorithm is introduced for the analysis of structures involving magnetized ferrites. A critical issue of time and space synchronism ensuring second order accuracy is discussed, and a method to provide it based on extrapolation rather than interpolation used by previous authors is presented. Numerical examples validating the method are given.**

I. Introduction

Recently, the versatility of FDTD has been further enhanced with the introduction of algorithms capable of solving electromagnetic problems that involve magnetized ferrites. Two distinct approaches have been reported.

In the first one, the relations describing ferrite—electromagnetic field interactions derived in the frequency domain—are exploited. Magnetic flux $\vec{B}$ is related to a magnetic strength vector $\vec{H}$ via a frequency dependent tensor of Polder $\overleftrightarrow{\mu}$. This relation, transformed to the time domain by means of the convolution theorem, is introduced into the Maxwell's equation system. Subsequent discretization yields the FDTD scheme. A successful implementation of this concept was described in details by Kunz *et al.* in their book [1], where issues of transforming convolutions into an efficient recursive technique and decoupling of the difference equations were also addressed.

The other approach goes back into the physics of ferrites. Maxwell's equations are supplemented with the Gilbert's equation of motion, which describes, in the time domain, the interactions between magnetic strength and magnetization vectors in magnetized ferrites. In the context of FDTD this idea was first reported by [2], [3]. As in the convolution approach, the discretization process is complicated by the existence of magnetized ferrites, and care must be taken to maintain second-order accuracy of central difference scheme. A properly synchronized in space and in time ferrite FDTD method was introduced by Pereda [4] and almost simultaneously in [5], [6]. Pereda and his co-workers later utilized the Rotated Richtmyer scheme [7] in their FDTD method, which allowed them to avoid spatial interpolation of vector $\vec{B}$, but at the expense of interpolation of vector $\vec{E}$.

In this letter, a further enhancement to ferrite FDTD techniques is introduced in which maintenance of the proper time synchronism within the entire FDTD scheme is simplified.

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II. Formulation

In the magnetized, saturated ferrite media, the electromagnetic wave is governed by the following set of partial differential equations:

$$\nabla \times \vec{E} = -\frac{d}{dt}\vec{B} \tag{1}$$

$$\nabla \times \vec{H} = \frac{d}{dt}\vec{D} \tag{2}$$

$$-\gamma(\vec{M} \times \vec{H}) = \frac{d}{dt}\vec{M} \tag{3}$$

where: γ is a gyromagnetic ratio, $\vec{M}=\vec{M}_s+\vec{m}$, $\vec{H}=\vec{H}_i+\vec{h}$. Vectors $\vec{M}_s$, $\vec{H}_i$ denote saturation magnetization and static biasing magnetic field, respectively, while $\vec{m}$ and $\vec{h}$ are time varying magnetization and magnetic strength vectors, respectively.

Assuming that $\vec{M}_s \gg \vec{m}$ and $\vec{H}_i \gg \vec{h}$, and knowing that $\vec{B} = \mu_0(\vec{H} + \vec{M})$, the following PDE system is obtained:

$$\frac{d}{dt}\vec{E} = \frac{1}{\epsilon}\nabla \times \vec{h} \tag{4}$$

$$\frac{d}{dt}\vec{h} = -\frac{1}{\mu_0}\nabla \times \vec{E} + \frac{d}{dt}\vec{m} \tag{5}$$

$$\frac{d}{dt}\vec{m} = -\gamma\,(\vec{m} \times \vec{H}_i + \vec{M}_s \times \vec{h}) \tag{6}$$

This system of equation needs to be discretized in order to derive an FDTD algorithm. As in the standard Yee's method, central differencing is applied to obtain an explicit, leap-frog scheme. Since the field components in the plane perpendicular to the biasing magnetization are coupled, care must be exercised in the discretization process in order to maintain the second order accuracy of the central difference scheme, i.e. a proper synchronization of the algorithm in space and in time has to be ensured. This issue will be addressed in the next section.

A. Synchronization in Space

To illustrate the synchronization problem the left sides of (4), (5) are discretized first:

$$\frac{\vec{E}^n - \vec{E}^{n-1}}{\Delta t} = \frac{1}{\epsilon}\left\{\nabla \times \vec{h}\right\}^{n-1/2} \tag{7}$$

$$\frac{\vec{h}^{n+1/2} - \vec{h}^{n-1/2}}{\Delta t} = -\frac{1}{\mu_o}\left\{\nabla \times \vec{E}\right\}^n - \left\{\frac{d}{dt}\vec{m}\right\}^n \tag{8}$$

where: n denotes the number of time steps elapsed since the beginning of the simulation.

Provided all the components are available in the specified moments of time, the above finite difference approximation introduces errors of order $o[(\Delta t)^2]$. To meet the specified time constraints, time derivative of $\vec{m}$ has to be computed using $\vec{h}^n$ (for the sake of clarity, it is assumed for the rest of this section that $H_i = 0$ and $\vec{M}_s = M_s \vec{a}_y$):

$$\left\{ \frac{d}{dt} \vec{m} \right\}^n = -\gamma \vec{M}_s \times \vec{h}^n \tag{9}$$

Unfortunately, $\vec{h}^n$ is not available. In [2], and also recently in [8], this fact was ignored and it was simply assumed that $\vec{h}^n = \vec{h}^{n-1/2}$. This allowed for a simple derivation of ferrite algorithm, but assured $o[\Delta t/2]$ accuracy only.

A more accurate solution is obtained if interpolation is used to fulfill time criterion in (9). It can be observed, that though $\vec{h}^n$ is not available, $\vec{h}^{n-1/2}$ is already known, and $\vec{h}^{n+1/2}$ is about to be determined. Simple linear interpolation yields:

$$\left\{ \frac{d}{dt} \vec{m} \right\}^n = -\frac{\gamma}{2} \left(\vec{M}_s \times \vec{h}^{n-1/2} + \vec{M}_s \times \vec{h}^{n+1/2} \right) \tag{10}$$

If this approximation is introduced into (8), a following pair of equations is obtained:

$$h_x^{n+1/2} - s h_z^{n+1/2} = h_x^{n-1/2} - \frac{\Delta t}{\mu_o} \vec{a}_x \cdot \{\nabla \times \vec{E}\}^n + s h_z^{n-1/2} \tag{11}$$

$$h_z^{n+1/2} + s h_x^{n+1/2} = h_z^{n-1/2} + \frac{\Delta t}{\mu_o} \vec{a}_z \cdot \{\nabla \times \vec{E}\}^n - s h_x^{n-1/2} \tag{12}$$

where: $s = \Delta t \frac{\gamma M_s}{2}$

Note, that these equations are coupled, therefore they need to be solved for $h_x^{n+1/2}$ and $h_z^{n+1/2}$ prior to the derivation of FDTD update formulae. Pereda, in [4], [7], successfully used an approach very similar to the one described above.

The numerical overhead associated with the decoupling in the formulation presented above can be avoided if extrapolation rather than interpolation is used as follows:

$$\vec{h}^n \approx \vec{h}^{n-1} + \left\{ \frac{d}{dt} \vec{h} \right\}^{n-1} \Delta t = \frac{1}{2} \left(3\vec{h}^{n-1/2} - \vec{h}^{n-3/2} \right),$$

which leads to the following approximation of the time derivative of $\vec{m}$:

$$\left\{ \frac{d}{dt} \vec{m} \right\}^n = -\frac{\gamma}{2} \vec{M}_s \times \left(3\vec{h}^{n-1/2} - \vec{h}^{n-3/2} \right) \tag{13}$$

Using this approximation, the following relations can be derived:

$$h_x^{n+1/2} = h_x^{n-1/2} - \frac{\Delta t}{\mu_0} \vec{a}_x \cdot \{\nabla \times \vec{E}\}^n + s\left(3h_z^{n-1/2} - h_z^{n-3/2} \right) \tag{14}$$

$$h_z^{n+1/2} = h_z^{n-1/2} - \frac{\Delta t}{\mu_0} \vec{a}_z \cdot \{\nabla \times \vec{E}\}^n + s\left(3h_x^{n-1/2} - h_x^{n-3/2} \right) \tag{15}$$

Unlike (11), (12), the above equations are not coupled and can readily be used to derive FDTD update formulae. Although much simpler, they provide second-order accuracy in time.

If $H_i \neq 0$ is allowed, it is not possible to directly compute h fields. Instead, magnetization has to be computed first, and then its time derivative must be used to update h fields in (8), as shown below:

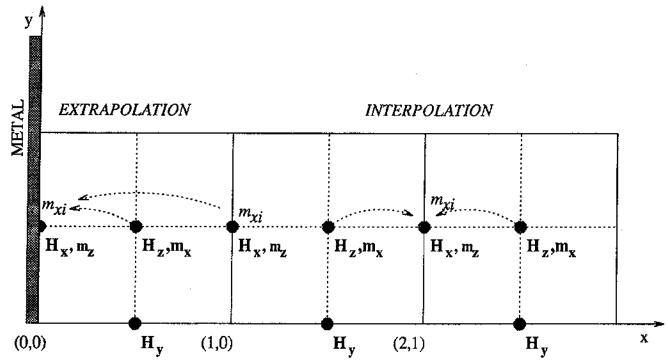

Fig. 1. Interpolation and extrapolation schemes in a Yee mesh modified for ferrite media.

$$m_x^{n+1/2} = m_x^{n-1/2} + \frac{\Delta t}{2} \gamma \Big[H_i (3m_z^{n-1/2} - m_z^{n-3/2}) - M_s (3h_z^{n-1/2} - h_z^{n-3/2}) \Big] \tag{16}$$

$$m_z^{n+1/2} = m_z^{n-1/2} - \frac{\Delta t}{2} \gamma \Big[H_i (3m_x^{n-1/2} - m_x^{n-3/2}) - M_s (3h_x^{n-1/2} - h_x^{n-3/2}) \Big] \tag{17}$$

$$h_x^{n+1/2} = h_x^{n-1/2} - \frac{\Delta t}{\mu_0} \vec{a}_x \cdot \{\nabla \times \vec{E}\}^n - \left(m_x^{n+1/2} - m_x^{n-1/2} \right) \tag{18}$$

$$h_z^{n+1/2} = h_z^{n-1/2} - \frac{\Delta t}{\mu_0} \vec{a}_z \cdot \{\nabla \times \vec{E}\}^n - \left(m_z^{n+1/2} - m_z^{n-1/2} \right) \tag{19}$$

B. Space Synchronism

As it is the case with time, space synchronism ensuring second-order accuracy is not automatically provided by the central differencing applied to the right hand side of (18), (19). This directly results from the coupling between magnetic fields perpendicular to the biasing magnetization and the organization of the modified Yee cell. For instance, magnetization component m_x, though required at the location of h_x, is actually evaluated at the location of h_z field component, due to the localized (in space) nature of the equation of motion (compare (16)). To alleviate this problem, interpolation is used (see Fig. 1), in a similar manner as in [4]. For example, in the case of 2D mesh of compact FDTD:

$$m_{xi}(ix, iy) = \frac{1}{2} [m_x(ix-1, iy) + m_x(ix, iy)] \tag{20}$$

where: standard FDTD notation is used to describe the location of vectors in space, and subscript i denotes interpolated values.

This interpolation scheme breaks down at the metal-ferrite interface. Extrapolation can be used instead, yielding, for instance ix=0:

$$m_{xi}(0, iy) = 2m_x(0, iy) - m_{xi}(1, iy)) \tag{21}$$

The interpolated values m_{*i} should be used in (18), (19) in place of m_*.

TABLE I
PHASE CONSTANTS OF FUNDAMENTAL MODE OF RECTANGULAR WAVEGUIDE FULFILLED WITH MAGNETIZED FERRITE. WAVEGUIDE: 22.86 x 10.16mm, FERRITE: M_s=2000Gs, H_i=0.1*M_s, ϵ_r=9; MESH: $\Delta x = a/10$

β[rad/m]	83.27	151.5	200.9	254.3	300.3	361.3	398.7	468.3	501.2
F[GHz]	6.630	6.911	7.207	7.604	8.003	8.601	9.001	9.799	10.20
errors [%]									
FDTD[4]	-0.69	-0.62	-0.55	-0.49	+2.05	+2.04	+4.26	+4.04	+3.94
FDTD this	-0.16	-1.85	-1.16	-0.42	+0.15	+0.36	+0.44	+0.69	+0.89

TABLE II
PHASE CONSTANTS OF FUNDAMENTAL MODE OF RECTANGULAR WAVEGUIDE PARTIALLY FILLED WITH MAGNETIZED FERRITE. WAVEGUIDE: 22.86 x 10.16mm, (FERRITE SLAB ADJACENT TO THE LATERAL WALL OF THE WAVEGUIDE): WIDTH $a/3$, M_s=2000Gs, H_i=0.1*M_s, ϵ_r=9; MESH: $\Delta x = a/100$

β[rad/m]	-500	-300	-200	-100	0	100	200	300	500
exact F[GHz]	10.57	7.669	6.696	6.618	6.828	7.376	8.188	9.128	11.32
FDTD error	+0.10	+0.39	+0.48	+0.65	+0.73	+0.26	-0.55	-0.49	-0.13

TABLE III
RESONANCE FREQUENCY OF A FUNDAMENTAL MODE IN A RECTANGULAR RESONATOR LOADED WITH CYLINDRICAL MAGNETIZED ROD. RESONATOR: 22.86 x 10.16 x 28.85mm. FERRITE RODS: FOR Φ=2 and 3mm — ϵ_f=13, M_s=1750Gs, H_i=0; for Φ=4mm — ϵ_f=13.5, M_s=950, H_i=0

Rod Φ [mm]	2	3	4
MM[9]	8.134	7.895	7.453
FDTD	8.136	7.906	7.435
measured [9]	8.118	7.851	7.442

III. NUMERICAL EXAMPLES

Both a 3D and a compact (2.5-D) versions of the algorithm were implemented. As a first test, phase constants of a rectangular waveguide fulfilled with magnetized ferrite media were calculated using exact formulae and compared in Table I, with the results generated by Pereda's FDTD code [4] and the technique presented in this paper. Although a rough mesh was used, both FDTD programs converged well to the exact results with slightly better accuracy shown by the method presented in this paper. It was observed that although such a rough mesh resolution is usually sufficient for homogeneously filled waveguides, to obtain a stable solution for more complicated structures higher densities of order $a/30 - a/100$ are required.

In Table II, the phase constants of a fundamental mode in a rectangular waveguide loaded with a slab of magnetized ferrite (Fig. 2(a)), obtained using exact formulae and FTDT, are compared. Again, the FDTD produced results compare very well with the theoretical ones.

As a final test, a rectangular resonator comprising a longitudinally magnetized ferrite rod of a circular cross-section (Fig. 2(b)) was considered. In Table III, the resonance frequencies computed using FDTD, Mode Matching, and measured experimentally are shown. Both numerical techniques produced results that are in a very good agreement with the experimental data.

IV. CONCLUSIONS

A more efficient ferrite FDTD algorithm was introduced for the analysis of structures involving magnetized ferrites. Time synchronism ensuring second-order accuracy was achieved by extrapolation rather than interpolation used by previous authors. This simplified the resulting FDTD update formulae and yielded higher computational efficiency. Both 3D and compact (for the analysis of waveguides) algorithms were implemented. Numerical examples included computations of simple structures for which exact formulae were known, and complicated ones for which the FDTD results were compared with data generated by mode matching technique and obtained experimentally. The method was found numerically stable. It should be pointed out, however, that rigorous study of stability of the ferrite FDTD has not been carried out yet.

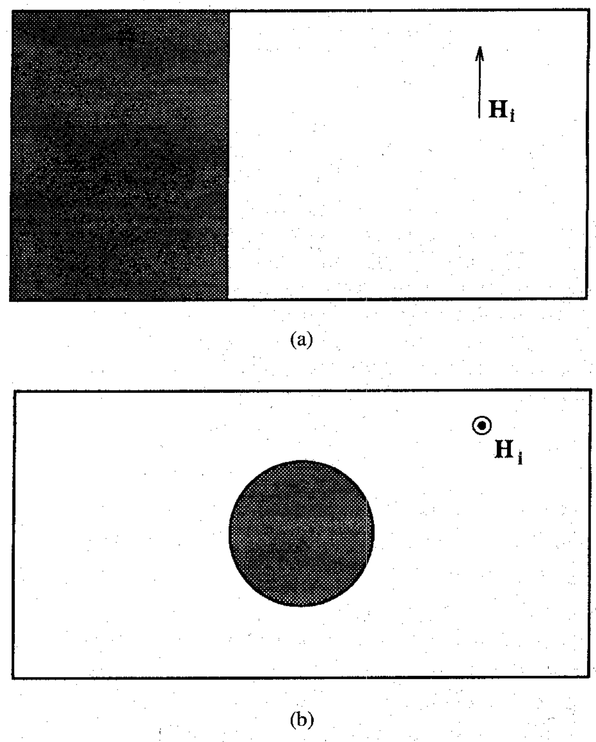

Fig. 2. Rectangular waveguides comprising magnetized ferrites. (a) Slab-loaded guide. (b) Rod-loaded guide.

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